Simple exercises related to models with heterogeneous agents.

Part A: Simulate a collection of rep. agent economies

Consider a unit mass of agents that all behave according to the neo-classical growth model (as in modelcloglinear.mod). They differ, however, in terms of their initial capital stock. In particular, suppose that the initial capital stock is uniformly distributed from 40% below to 40% above the steady state capital stock. Write the algorithm to simulate this collection of economies.

1. Use dynare to obtain the individual policy rule
2. Draw a long sequence of shocks and construct a time series for z_t.
3. Construct the grid with the initial histogram
4. Now start iterating. You can do exactly like we did in class except that every period the policy function changes with the value of z_t. Note that the cross-sectional heterogeneity will eventually disappear in this example

Part B: A bit more simulation

Consider the model from the slides (simple heterogeneous agents model without aggregate uncertainty) and in particular consider the following solution

$$k_{i,t} = 205.93 + 0.98\left(k_{i,t-1} - 205.93\right) - 0.38 \ \mathit{if} \ \ e_{i,t} = -0.1$$

$$k_{i,t} = 205.93 + 0.98\left(k_{i,t-1} - 205.93\right) + 0.38 \ \mathit{if} \ \ e_{i,t} = +0.1$$

1. Calculate the value the agent's capital stock would converge to if he would get the bad draw for $e_{i,t}$ for ever and the value when he would get the good draw for ever. This will give the lower and the upper bound of the distribution.
2. Construct a grid for the distribution. Take 100 grid points for capital. Since we have two values for e this means we have 200 grid points. Let the initial mass at each grid point be equal to 0.005. Use equidistant grid points. Use the given policy rules to iterate on the distribution. Each period, go through all the grid point and reallocate the mass at this grid point to next period's distribution. This requires the following steps.
    a. Use the policy rule to determine next period's capital stock.
    b. Determine the gridpoint just below and just above this value. With equidistant gridpoint this can be found by i^* = floor(k-k_{low})/step, where k is the chosen capital stock, k_{low} is the lower bound of the grid, step the stepsize, and floor(x) is a matlab command that gives the highest integer value less than x. Now figure out how to allocate the mass of the grid point you are looking at to the i^*-th and i^* -th grid points. Don't forget that half of the agents switch employment status.

Part C: Solve for the equilibrium *r* in a model with heterogeneous agents without aggregate uncertainty

Use the model from the slides (simple heterogeneous agents model) and solve for the equilibrium K

1. Write a Dynare program to solve the individual problem for which the equilibrium interest rate can be read as a parameter from an outside file
2. Write a Matlab routine that iterates on the values for *r*. Thus,
    a. Start with a guess for *r*
    b. Run the Dynare program to get the individual policy rules
    c. Check the equilibrium condition, that is, check whether total savings is equal to zero at the specified interest rate. Since all agents are identical you can either check whether the amount of savings aggregated across agents is equal to zero or whether the average amount of savings of one agent over a long time period is equal to zero
    d. Adjust r
3. Do this program using a linear and a log-linear solution for the policy rule. eqrloglinear.mod file is the mod file for the linear case.

Part D: Heterogeneous agents and aggregate uncertainty

The file simpleheteroB.mod solves the individual problem for a given law of motion for aggregate capital, *K*. The goal is to update this law of motion so that we have an equilibrium.

To do
1. Solve for the aggregate capital stock, *K*, using Xpa. Thus, solve the individual policy rule and directly obtain an updated aggregate policy rule from the individual policy rule. Now repeat until convergence.
2. Fix the solution for *K* found in step 1. For this law of motion obtain a log-linear solution of the individual problem. Simulate a cross-section using only this log-linear solution and check how the time series for the cross-sectional mean of capital differs from the one generated by the linear law of motion.
3. Repeat step but now use the KS algorithm to update. Use both stochastic and non-stochastic simulation.