1 Matching model

One model that would be good to solve with different procedures is the following matching model (with capital). Try not to modify the equations but stick to the formulation of the model and equations as they are given here. This will make it easier to compare programs.

Employment flows n_t is the number of workers that are productive in period $t+1$. It consists of those that were productive in period t and did not face an exogenous separation (at the end of period t) plus the number of matches made in period t. The number of matches in period t is determined by the number of unemployed, $u_t = 1 - n_{t-1}$ and the number of vacancies, v_t.

$$n_t = (1-\rho^x)n_{t-1} + \phi_0 u_t^{\phi_1} v_t^{1-\phi_1} \tag{1}$$

$$u_t = 1 - n_{t-1} \tag{2}$$

Euler equation Household optimization leads to the standard Euler equation

$$c_t^{-\eta} = \beta \mathrm{E}_t \left[c_{t+1}^{-\eta}(r_{t+1} + 1 - \delta) \right] \tag{3}$$

where the rental rate of capital is given by the marginal product of capital

$$r_t = \alpha z_t \left(\frac{k_{t-1}}{n_{t-1}} \right)^{\alpha - 1} \tag{4}$$

Firm profits and firm value Firms rent capital on a spot market and they maximize profits. The wage rate is constant. The optimization problem is thus given by

$$p_t = \max_{\widehat{k}_t} z_t \widehat{k}_t^{\alpha} - r_t \widehat{k}_t - w$$

Using the equilibrium condition 4 we can write the solution of this problem as

$$p_t = (1-\alpha) z_t \left(\frac{k_{t-1}}{n_{t-1}} \right)^{\alpha} - w \tag{5}$$

Firm value can then be written as

$$g_t = \beta \mathrm{E}_t \left[p_{t+1} + (1-\rho^x) g_{t+1} \right] \tag{6}$$

Free entry The posting cost, χ, has to be equal in equilibrium to the expected benefit

$$\chi = \phi_0 \left(\frac{u_t}{v_t}\right)^{\phi_1} g_t \tag{7}$$

Output In period t there are n_{t-1} productive relationships. Aggregate capital available at the beginning of t is equal to k_{t-1}. The amount of capital in each relationship is, thus, equal to k_{t-1}/n_{t-1}. Aggregate output is defined as

$$o_t = n_{t-1}\left(z_t\left(\frac{k_{t-1}}{n_{t-1}}\right)^{\alpha}\right)$$

or

$$o_t = z_t k_{t-1}^{\alpha} n_{t-1}^{1-\alpha} \tag{8}$$

Aggregate budget constraint Society's budget constraint is given by

$$c_t + k_t = o_t + (1-\delta)k_t \tag{9}$$

The first term on the right-hand side is equal to the sum of rental payments, wage payments, and the income of the entrepreneur, but it is easier to work with this expression.

Notation Note we have 9 equations in 9 unknowns, n_t, g_t, v_t, c_t, k_t, n_t, r_t, u_t, p_t, and o_t. Two equations are expectational equations. Using the notation of Uhlig we have

$$x_t = \begin{bmatrix} k_t \\ n_t \end{bmatrix}$$

and

$$y_t = \begin{bmatrix} c_t \\ r_t \\ u_t \\ v_t \\ g_t \\ p_t \\ o_t \end{bmatrix}$$

Parameter values

parameter	value
β	0.96
α	0.36
δ	0.025
η	1
ψ	0.95
σ_ε	0.00712
ϕ_1	0.5
χ	0.7

There are two additional parameters, namely ϕ_0 and w. If we know the steady-state values of all endogenous variables, then we do not need these two variables to obtain a linear solution. So instead of assigning a value to these two parameters directly, we define them implicitly by two steady-state relationships. The first is that—in steady state—the matching probability of the worker, λ^w, is equal to 0.71. This trick of assigning steady-state values instead of parameter values can make calculating steady state values very easy. For example, we now get directly that the steady state value for employment is given by

$$n = \frac{\lambda^w}{\lambda^w + \rho^x}.$$

This directly gives the steady-state value for unemployment. Then, if one has the steady state value for the ratio of vacancies to unemployment one can calculate (if one would want to) ϕ_0. The second steady state relationship says that the entrepreneur gets 10% of the revenues minus interest payments. That is

$$p = 0.10(1-\alpha)\left(\frac{k}{n}\right)^{\alpha}.$$

It is easy to see that this implies a value for w, but again you do not need w. With p you can do the linearization.