1 Matching model for dynare example

1.1 Households

A household consists of a continuum of households, each of which is either employed or unemployed. Employed members get a (fixed) wage w. Unemployed members are searching for a new job. There is no disutility of work or search and all unemployed workers accept a job if they get offered one. Households own the capital stock which they rent out to firms. The maximization problem of the household is given by

$$\begin{aligned} v(k_{t-1}, n_{t-1}, z_t) &= \max_{k_t, c_t} \frac{c_t^{1-\eta} - 1}{1-\eta} + \beta \mathrm{E}_t \left[v(k_t, n_t, z_{t+1}] \right. \\ \text{s.t.} \quad c_t + k_t &= n_{t-1} w + r_t k_{t-1} + (1-\delta) k_{t-1} + f_t \end{aligned}$$

Here f_t is equal to the profits the household receives from the firms. The other symbols have their typical interpretation and are summarized in the table below. Note that the household doesn't choose employment. It (re)supplies a unit mass each period but how many are hired depends on the number of vacancies being posted.

1.2 Firms

The production of the representative firm is given by

$$y_t = k_{t-1}^{\alpha} n_{t-1}^{1-\alpha}.$$

Capital is rented on a spot market. A fixed fraction, ρ^x, of employment separates each period. Firms can find new workers by posting vacancies at cost ζ. The representative firm takes the matching probability, λ_t^f, as given. The firm's maximization problem is given by

$$\begin{aligned} w(n_{t-1}, z_t) &= \max_{n_t, v_t} c_t^{-\eta} \left(k_{t-1}^{\alpha} n_{t-1}^{1-\alpha} - w n_{t-1} - r_t k_{t-1} - \zeta v_t \right) + \beta \mathrm{E}_t \left[w(n_t, z_{t+1}) \right] \\ \text{s.t.} \quad n_t &= (1-\rho^x) n_{t-1} + \lambda_t^f v_t \end{aligned}$$

Note that the firm evaluates profits using the marginal utility of the household (its owner) which it takes as given

1.3 Matching and equilibrium

Matches are equal to

$$\phi_0 u_t^{\phi_1} v_t^{1-\phi_1},$$

where

$$u_t = 1 - n_{t-1}$$

The matching probability for the firm is, thus, given by

$$\lambda_t^f = \phi_0 \left(\frac{u_t}{v_t} \right)^{\phi_1}$$

1.4 Definition of variables

n_t	end-of-period t employment
k_t	end-of-period t capital
p_t	marginal profit of one extra worker (mp-wage)
g_t	value of match
c_t	consumption
u_t	unemployed workers in period t
v_t	vacancies posted in t
r_t	rental rate of capital
y_t	output
z_t	productivity
e_t	innovation of capital

1.5 Equations of the model

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\begin{aligned}
n_t &= (1-\rho^x)n_{t-1} + \phi_0 u_t^{\phi_1} v_t^{1-\phi_1} \\
g_t &= \beta \mathrm{E}_t \left[p_{t+1} + (1-\rho^x) g_{t+1} \right] \\
\zeta &= \phi_0 \left(\frac{u_t}{v_t} \right)^{\phi_1} g_t \\
y_t &= c_t + k_t - (1-\delta)k_{t-1} + \zeta v_t \\
c_t^{-\eta} &= \beta \mathrm{E}_t \left[c_{t+1}^{-\eta} \left(r_{t+1} + 1 - \delta \right) \right] \\
r_t &= \alpha z_t \left(\frac{k_{t-1}}{n_{t-1}} \right)^{\alpha-1} \\
u_t &= 1 - n_{t-1} \\
p_t &= (1-\alpha) z_t \left(\frac{k_{t-1}}{n_{t-1}} \right)^{\alpha} - w \\
y_t &= z_t k_{t-1}^{\alpha} n_{t-1}^{1-\alpha} \\
\ln(z_t) &= \psi \ln(z_{t-1}) + e_t
\end{aligned}
$$

1.6 Dynare Equations

To get the dynare equations you only have to delete the conditional expectation. Dynare "knows" that there must be an expectational operator (and prediction error) in these equations because they have "$t+1$" terms

1.7 Comments

- variables known at the beginning of t must be dated $t-1$