Simple model with heterogeneous agents

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Individual agent

- Subject to i.i.d. idiosyncratic productivity shocks $e_{i,t} \in \{0.9, 1.1\}$
- Incomplete markets
  - only way to save is through holding capital
  - investing small and large amounts "costlier" than intermediate amounts

Individual agent

$$\max_{\{c_{i,t},k_{i,t+1}\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t \ln(c_{i,t}) - \frac{\zeta_1}{\zeta_0}\exp(-\zeta_0 k_{i,t})$$
$$\text{s.t.} \quad c_{i,t} + k_{i,t} = r_t k_{i,t-1} + w_t e_{i,t} + (1-\delta)k_{i,t-1}$$

$$-\frac{1}{c_{i,t}} + \zeta_1 \exp(-\zeta_0 k_{i,t}) + \mathrm{E}_t\left[\frac{\beta}{c_{i,t+1}}(r_{t+1}+1-\delta)\right] = 0$$

Equilibrium

- Unit mass of worker, $L_t = 1$
- Competitive firm so agent faces competitive prices
  - $w_t = (1 - \alpha) K_t^{\alpha}$
  - $r_t = \alpha K_t^{\alpha - 1}$
- No aggregate risk so

$$K_t = K$$

- How to find the equilibrium K?

Algorithm

- Guess a value for r
- This implies values for K and w
- Solve the individual problem with this value for r (and K & w)
- Simulate the economy and calculate the implied value for K, K^{imp}
- If $K^{\text{imp}} < K$ then r too low so raise r, say

$$r^{\text{new}} = r + \lambda(K - K^{\text{imp}})$$

- Iterate until convergence

Stochastic aggregateproductivity

- $w_t = (1 - \alpha) z_t K_t^{\alpha}$
- $r_t = \alpha z_t K_t^{\alpha - 1}$
- $z_t = 1 - \rho + \rho z_{t-1} + \varepsilon_t$

State variables

- Individual: $k_{i,t}$, $e_{i,t}$
- Aggregate: z_t and cross-sectional distribution

Just suppose

- Just suppose aggregate law of motion for K_t is given by

$$K_t = a_0 + a_1 K_{t-1} + a_2 z_t$$

  and coefficients are known
- How would you solve for the individual problem?

First-order conditions

$$c_{i,t} + k_{i,t} = r_t k_{i,t-1} + w_t e_{i,t} + (1-\delta) k_{i,t-1}$$

$$-\frac{1}{c_{i,t}} + \zeta_1 \exp(-\zeta_0 k_{i,t}) + \mathrm{E}_t \left[\frac{\beta}{c_{i,t+1}} \left(r_{t+1} + 1 - \delta \right) \right] = 0$$

Use:

$$\begin{aligned} r_t &= \alpha z_t K_t^{\alpha-1} \\ r_{t+1} &= \alpha z_{t+1} K_{t+1}^{\alpha-1} = \alpha z_{t+1} \left(a_0 + a_1 K_t + a_2 z_{t+1} \right)^{\alpha-1} \end{aligned}$$

Standard problem; could be solve with Dynare

How to get law of motion for

K_t?

- Simulate using a finite number of agents and random draws for z_t *and* e_{it}
- Simulate using a continuum of agents and random draws for z_t
- Explicit aggregation

Explicit aggregation

Individual policy rule:

$$k_{i,t} = b_0 + b_1 k_{i,t-1} + b_2 e_{i,t} + b_3 z_t + b_4 K_{t-1}$$

Implied aggregate policy rule:Individual policy rule:

$$\begin{aligned} K_t &= b_0 + b_1 K_{i,t-1} + b_3 z_t + b_4 K_{t-1} \\ &= b_0 + (b_1 + b_4) K_{t-1} \end{aligned}$$

Log-linear individual problem

$$\exp(c_{i,t}) + \exp(k_{i,t}) = r_t \exp(k_{i,t-1}) + w_t e_{i,t} + (1-\delta)\exp(k_{i,t-1})$$

$$-\exp(-c_{i,t}) + \zeta_1 \exp(-\zeta_0 \exp(k_{i,t})) + \mathrm{E}_t\left[\beta \exp(-c_{i,t+1})(r_{t+1} + 1 - \delta)\right] = 0$$

Use:

$$\begin{aligned} r_t &= \alpha z_t K_t^{\alpha-1} \\ r_{t+1} &= \alpha z_{t+1} K_{t+1}^{\alpha-1} = \alpha z_{t+1}\left(a_0 + a_1 K_t + a_2 z_{t+1}\right)^{\alpha-1} \end{aligned}$$

Algorithm

- Get aggregate law of motion for linear individual policy rules
- Get log-linear individual policy rules taking aggregate law of motion as given
- Note that linear individual policy rules are linear first-order approximation of log-linear individual policy rules