Accuracy of models with heterogeneous agents

Wouter J. Den Haan

University of Amsterdam

February 23, 2010

Introduction

Models with heterogeneous agents have many different dimensions
Krusell-Smith algorithm

- Numerical integration
  - Typically not a source of inaccuracy (if you use quadrature)
  - Use more quadrature nodes and see whether results change
- Accuracy of individual policy rule (given aggregate rule)
  - Use Euler equation errors (like plugging in your solution and checking errors are zero)
- Accuracy of aggregate policy rule (given individual rule)
- Error accumulation from one block to the other

Accuracy of aggregate law of motion

- Important to check its accuracy *without imposing functional form assumption*
- Despite its disadvantages a simulation of the complete economy with an accurate method seems a good idea
- So what are the inputs?
  - individual policy rule (fixed)
  - initial cross-sectional distribution over capital and employment status
  - an accurate algorithm to simulate the economy
  - a candidate aggregate law of motion which needs to be checked

$$m_{t+1} = \bar{\phi}(z_{t+1}, z_t, m_t; \widehat{\bar{\alpha}}) + u_{t+1}$$

-
  - $u_{t+1} \equiv 0$ for true transition law

Popular accuracy procedure

- Simulate a time series $\{m_t\}_{t=1}^{T}$ using *only* individual policy rules
- Use those values for m_t on LHS and RHS and check error
- (note that KS simulataneously estimates $\bar{\alpha}$ but this is not necessary
- Accuracy measuer is the R^2 (and the standard error of the regression)

Problems

- Overfitting (adding higher-order terms can only improve your accuracy measures)
- You shouldn't use the same draw to estimate $\bar{\phi}(\cdot)$ and evaluate it
- R^2 and $\widehat{\sigma}_u$ are averages, that is a weak metric
- R^2 scales. That is R^2 of

$$\Delta m_{t+1} = m_{t+1} - m_t = \bar{\phi}(z_{t+1}, z_t, m_t; \widehat{\bar{\alpha}}) - m_t + u_{t+1}$$

is substantially lower than that of

$$m_{t+1} = \bar{\phi}(z_{t+1}, z_t, m_t; \widehat{\bar{\alpha}}) + u_{t+1}$$

- Not clear what is a bad R^2

But the real problem is:

- data generated by the "true" law of motion is used as the explanatory variable in the approximating law of motion

New accuracy procedure

1. Generate series independently (using only same aggregate shocks and initial distribution)
   1. As above generate a time series $\{m_t\}$ by simulating
   2. *Without using this time series* generate a new series using the candidate aggregate law of motion
2. Calculate the max (and check at what kind of observation it occurs)
3. Plot both time series (essential accuracy plot)
4. Compares some properties of the two laws of motion (e.g. impulse response functions)

What did Krusell & Smith use?

- True, they did emphasize the R^2 and $\hat{\sigma}_u$
- But they also looked at many other measues
  - alternative functional forms
  - economic arguments
  - *100 period-ahead forecast errors* (This turns out to be just as powerful as my max)

What does a modification of the KS law of motion do to the R^2?

$$\ln K_t = \alpha_1 + \alpha_2 a_t + \alpha_3 \ln K_{t-1} + u_t$$

$$\alpha_3 = 0.96404$$
$$R^2 = 0.99999729$$

Experiment:

- Change α_3
- Adjust α_1 to keep mean of u_t equal to zero
- Recalculate R^2
- Calculate implied standard deviation of $\ln K_t$ to evaluate magnitude of the change

	R^2	implied standard deviation
$\alpha_3 = 0.9604$ (original regression)	0.99999729	0.0248
$\alpha_3 = 0.954187$	0.99990000	0.0217
$\alpha_3 = 0.9324788$	0.99900000	0.0174
$\alpha_3 = 0.8640985$	0.99000000	0.0113

Monte Carle example #1

Truth is given by

$$m_{t+1} = \alpha_0 + \alpha_1 m_t + \alpha_2 a_t + \alpha_3 m_{t-1}$$

Approximating law of motion

$$m_{t+1} = \bar{\alpha}_0 + \bar{\alpha}_1 m_t + \bar{\alpha}_2 a_t$$

Monte Carlo example #2

Truth is given by

$$\begin{aligned} m_{t+1} &= \alpha_0 + \alpha_{1,t} m_t + \alpha_2 a_t. \\ \alpha_{1,t} &= \left(\alpha_1 + \frac{\alpha_3}{\alpha_4 \exp(-\alpha_5 m_t)} \right) \end{aligned}$$

Approximating law of motion

$$m_{t+1} = \bar{\alpha}_0 + \bar{\alpha}_1 m_t + \bar{\alpha}_2 a_t$$

Table 1: Parameter Values

Parameter	Experiment #1		Experiment #2	
	#1.1	#1.2	#2.1	#2.2
α_0	0	0		0
α_1	1.08	1.38	0.65	0.65
α_2	1	1	1	1
$\alpha 3$	-0.1	-0.4	0.3	0.3
α_4	-	-	0.01	0.01
α_5	-	-	50	50
ρ_0	0	0	0	0
ρ_1	0	0	0.95	0
σ	0.00472	0.15436	$6.3891 * 10^{-4}$	$8.616 * 10^{-3}$

Notes: All parameter sets imply a standard deviation for the underlying series equal to 2.5%.

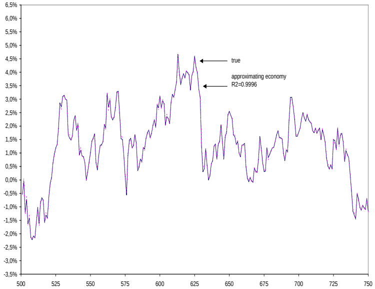
Traditional accuracy test

	#1.1	#1.2	#1.3	#1.4
R^2	0.9995	0.9940	0.99983	0.99981

minimum across Monte Carlo replications (that is, there are even higher ones)

Figure 1: In-sample fit of approximating law of motion

Panel A: Experiment 1.1



Panel B: Experiment 1.2

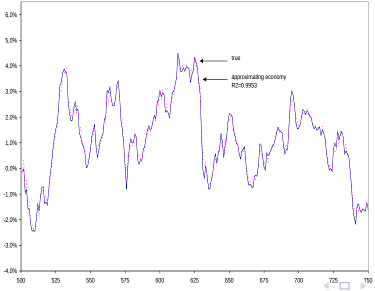
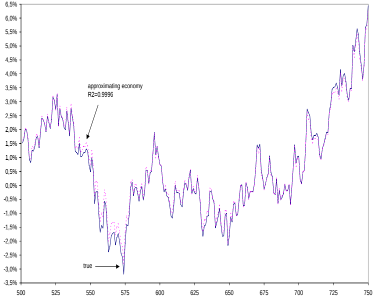


Figure 2: The essential accuracy plot – Separately generated series

Panel A: Experiment 1.1



Panel B: Experiment 1.2

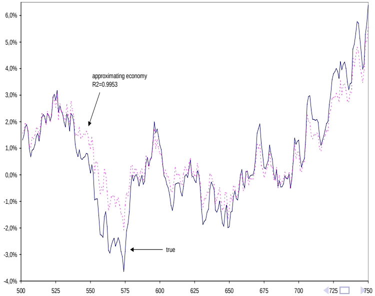
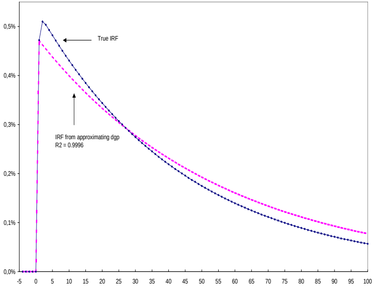


Figure 3: Impulse response functions

Panel A: Experiment 1.1



Panel B: Experiment 1.2

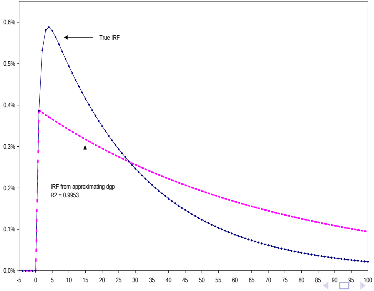
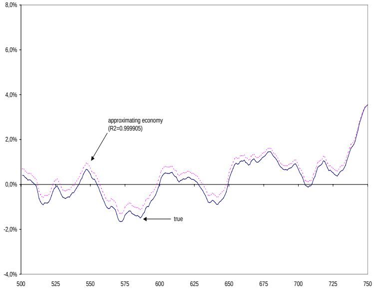


Figure 4: The essential accuracy plot – Separately generated series

Panel A: Experiment 2.1



Panel B: Experiment 2.2

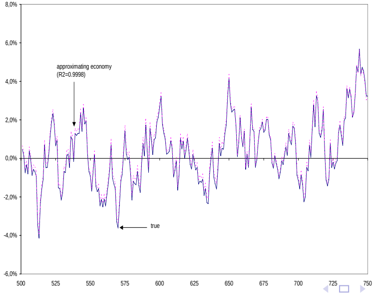


Figure 5: The essential accuracy plot – Separately generated series
Experiment 2.2 – part of simulation where maximum error occurs

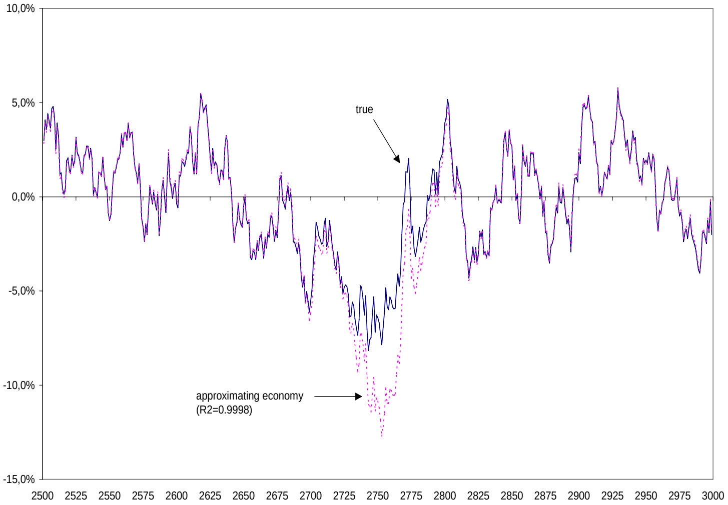
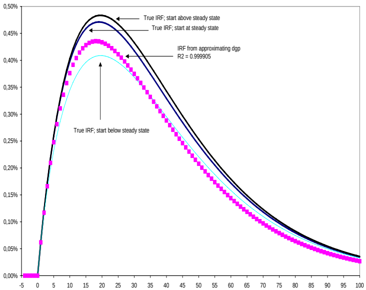


Figure 6: Impulse response functions

Panel A: Experiment 2.1



Panel B: Experiment 2.2

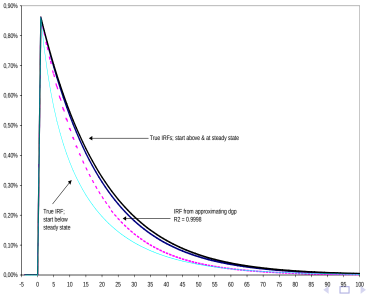


Figure 7: The essential accuracy plot – Separately generated series

Krusell-Smith economy

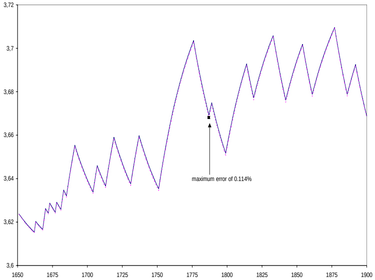
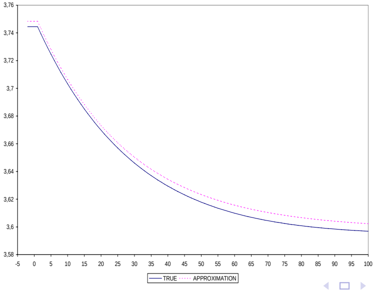


Figure 8: Impulse response function in the KS economy



New accuracy test

	#1.1	#1.2	#1.3	#1.4
$\widehat{u}^{max}$	0.83%	3.34%	1.86%	1.83%
$\widehat{u}^{ave}$	0.16%	0.67%	0.11%	0.12%

minimum across Monte Carlo replications, that is, even in the best Monte Carlo are the errors not that small