

Dynamic programming and solution methods – 2007/08

Tinbergen Field Course

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1. a Consider the following correspondences defined on the interval $X = [0,2]$

i) $\Gamma(x) = \{ y \in [5,6] \cup [7,9] \}$

ii) $\Gamma(x) = \{ y \in (5,6) \}$

iii) $\Gamma(x) = \{ y \in [1,6] \}$ for $x \leq 1$

$\Gamma(x) = \{ y \in [1,2] \cup [2,3] \}$ for $x > 1$

Determine which correspondences are uhc and which correspondences are lhc.

1.b Consider the following correspondences on the set $X = [-1,1]$

i) $\Gamma(x) = \{ y \in [5,6] \cup [7,9] \}$

ii) $\Gamma(x) = \{ y \in (5,6) \}$

iii) $\Gamma(x) = \{ y \in [0, x^2+5] \}$

iv) $\Gamma(x) = \{ y \in [0, -x^2+5] \}$

Determine which correspondences are convex valued and which correspondences are compact valued. As always explain your answer.

1.c Give an example of a discontinuous function on a compact set that doesn't attain a maximum.

2. a What is the sup norm and what is the L^2 norm?

b. Can polynomials approximate the following (continuous but non-differentiable) function arbitrarily well? Does your answer depend on whether you use the sup norm or the L^2 norm?

$$f(x) = 0 \quad \text{for } x \leq 0$$

$$f(x) = x \quad \text{for } x > 0$$

3. Discuss the relationship between the sequence of functions $g_n: X \rightarrow X$ converging uniformly and converging in the sup norm.